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Super Harmonic Mean Labeling Of Joins Of Square Of Path Graph

S.Sriram[1], Dr.R.Govindarajan[2]
[1]*Assistant Professor, Department of Mathematics, Patrician College of Arts and Science, Adyar, Chennai[1],Head*
[2]*Associate Professor(Retd), P.G and U.G Department of Mathematics, D.G Vaishnav College , Arumbakkam, Chennai[2]*
Email:sanksriram@gmail.com[1]

Abstract- Let G= (V, E) be a graph with p vertices and q edges. A graph G={V, E} with p vertices and q edges is said to be a super Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels by an injective function f(x) from 1,2, … p+q in such a way that when each edge e=uv is labelled with $f(e=uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ and if f is such that $f\left(V\left(G\right)\right) \cup \left\{ f\left(e\right)/e \in E\left(G\right) = 1,2,...p+q \right\}$. In this case f is called Super Harmonic mean labelling of G. In this paper we have identified Square of Path graph $P_n^{\ 2}$ and attached an edge to form a join to the square of path graph $P_n^{\ 2}$ and proved that the join of square of path graph $P_n^{\ 2}$ is Super harmonic mean labelling and also have exhibited some important results connecting the joins of square of path graph $P_n^{\ 2}$.

Keywords- Square of Path graph $P_n^{\ 2}$,Super Harmonic Mean labelling, Super Harmonic Mean graph, Joins of Square of path graph $P_n^{\ 2}$

1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian J. A [1] has given a dynamic survey of graph labelling. The origin of graph labellings can be attributed to Rosa. The vertex set is denoted by V(G) and the edge set is denoted by E(G). Mean Labelling of graphs is introduced by .Somasundaran S and Ponraj R [2] and Sandhya S S, .Somasundaram S and Ponraj.R [3,4,5]introduced Super Harmonic mean labelling of graphs and also worked on harmonic mean labelling of cycle related graphs. Motivated towards the labelling of Super Harmonic mean labelling of graphs we have identified the Square of path graph $P_n^{\ 2}$ tried to attach edge to the square of path $P_n^{\ 2}$ graph to form a chain of square of path graph $P_n^{\ 2}$ which we call as Join of square of path $P_n^{\ 2}$ graph.. In this paper we intend to prove that the Join of square of path $P_n^{\ 2}$ graph is Super Harmonic Mean labelling graph and we exhibit some important results connecting the joins of a square of path $P_n^{\ 2}$ graph. The concepts that we refer to graph labelling is from handbook of graph theory [6].

2. PRELIMINARIES

Definition 2.1: A graph G= {V, E} with p vertices and q edges is said to be a super Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels by an injective function f(x) from 1,2, … p+q in such a way that when each edge e=uv is labelled with $f(e=uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ and if f is such that $f\left(V\left(G\right)\right) \cup \left\{ f\left(e\right)/e \in E\left(G\right) = 1,2,...p+q \right\}$.

Definition .2.2: For a path graph P_n with vertices $u_1, u_2, ...u_n$ the square of path $P_n^{\ 2}$ graph with n vertices $u_1, u_2, ...u_n$ and 2n-3 edges

Definition .2.3 : For a path graph P_n with vertices $u_1, u_2, ...u_n$ the square of path $P_n^{\ 2}$ graph with n vertices $u_1, u_2, ...u_n$ and 2n-3 edges we attach an edge to each of square of path $P_n^{\ 2}$ graph with another square of path $P_n^{\ 2}$ graph and define it as 1-join square of path $P_n^{\ 2}$ graph.

Note: The number of vertices of 1-join square of path P_n^2 graph has 2n vertices and 4n-5 edges

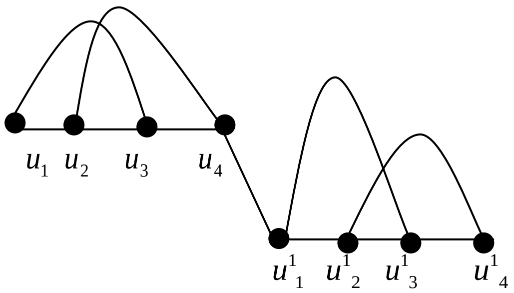


Figure.1: 1-Join square of path P_4^2

The above 1-join of square of path P_4^2 graph with 4 vertices and 5 edges attached by an edge to one square path graph P_4^2 with another square path graph P_4^2. Similarly we can construct 2-join square of path by attaching one more edge with another square path graph P_4^2. Continuing this way we get a chain of M-join of square of path P_4^2 graph. Similarly we can construct for any square of path P_n^2 graph and can call it as M-join of square of path P_n^2 graph.

Note : We here exclusively study the case of M-Join of square of path P_n^2 graph attached by edges $e_1, e_2, ..., e_n$ respectively one each to one square of path P_n^2 graph to square of path P_n^2 graph where n is of same length . So that the number of edges connected in each of square of path P_n^2 is maintained to be the same.

3. MAIN RESULTS

Theorem .3.1: The 1- Join square of path P_n^2 graph is a Super Harmonic Mean graph.

Proof: Let G=1-Join of square of path P_n^2 graph

Let us prove that G is a Super Harmonic Mean graph

Let us prove the theorem by labelling the vertices of the graph G.

We have the number of vertices in square of path P_n^2 graph is n and the number of edges is 2n-3. Now adding one edge between the square of path P_n^2 graph with another square of path P_n^2 graph we have the number of vertices in 1-Join of square of path P_n^2 graph is 2n and the number of edges in 1-Join of square of path graph is 4n-5.

Now let us label the vertices of 1-join of square of path P_n^2 graph as follows. We know that the first square of path P_n^2 has n vertices and second square of path P_n^2 has n vertices and totally 2n vertices is to be labelled.

Let us denote the Vertex Set of first square of path P_n^2 graph as $V = \{u_1, u_2, u_3, u_4, ...u_n\}$ and the vertex set of second square of path P_n^2 graph as $V^1 = \{u^1{}_1, u^1{}_2, u^1{}_3, u^1{}_4 ... u^1{}_n\}$. The edge set of first square of path P_n^2 graph is $E = \{e_1, e_2, e_3, e_4, ...e_{n-1}\} \cup \{e_{ij}, 1 \le i \le n-2, j = i+2\}$ and the edge of the second square of path P_n^2 Graph is $E^1 = \{e_1^1, e_2^1, e_3^1, e_4^1, ...e^1{}_{n-1}\} \cup \{e^1{}_{ij}, 1 \le i \le n-2, j = i+2\}$

Now by adding one edge between the first and second square of path P_n^2 graph we have the vertex set of 1-Join square of path P_n^2 graph as

$V(G) = \{u_1, u_2, u_3, u_4, ...u_n\} \cup \{u^1{}_1, u^1{}_2, u^1{}_3, u^1{}_4 ... u^1{}_n\}$

and the edge set as

$E(G) = \{e_1, e_2, e_3, e_4, ...e_{n-1}\} \cup \{e_1^1, e_2^1, e_3^1, e_4^1, ...e^1{}_{n-1}\}$

$\cup \{e_{ij}, e^1{}_{ij}, 1 \le i \le n-2, j = i+2\} \cup \{e\}$

The edges that are connecting between the first and second square of path P_n^2 graph are known as

$e_i = u_i u_{i+1}$ for $1 \le i \le n-1$

$e_i^1 = u_i^1 u_{i+1}^1$ for $1 \le i \le n-1$

$e_{ij} = u_i u_j$ for $1 \le i \le n-2$ and for j=i+2

$e_{ij}{}^1 = u_i{}^1 u_j{}^1$ for $1 \le i \le n-2$ and for j=i+2

$e = u_n u_1^1$

Now let us label the vertices of first square of path P_n^2 graph in correspondence to the vertices of second square of path P_n^2 graph as follows

$f(u_i) = 2i - 1$ for $1 \le i \le n$

$f(u^1{}_j) = 2(n+j) + 1$ for $1 \le j \le n$

Now we can compute the induced edge labelling as follows

$$f^*(e_i) = f^*(u_i u_{i+1}) = \left\lceil \frac{2 f(u_i) f(u_{i+1})}{f(u_i) + f(u_{i+1})} \right\rceil \text{or}$$

$$\left\lfloor \frac{2 f(u_i) f(u_{i+1})}{f(u_i) + f(u_{i+1})} \right\rfloor$$

$$f^*(e_i^1) = f^*(u^1_i u^1_{i+1}) = \left\lceil \frac{2f(u^1_i)f(u^1_{i+1})}{f(u^1_i)+f(u^1_{i+1})} \right\rceil$$

or $\left\lfloor \dfrac{2f(u^1_i)f(u^1_{i+1})}{f(u^1_i)+f(u^1_{i+1})} \right\rfloor$

$$f^*(e_{ij}) = f^*(u_i u_j) = \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil$$

or $\left\lfloor \dfrac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rfloor$

$$f^*(e_{ij}^1) = f^*(u_i^1 u_j^1) = \left\lceil \frac{2f(u_i^1)f(u_j^1)}{f(u_i^1)+f(u_j^1)} \right\rceil \text{ or}$$

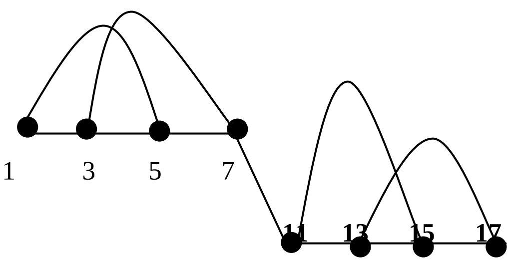
$$\left\lfloor \frac{2f(u_i^1)f(u_j^1)}{f(u_i^1)+f(u_j^1)} \right\rfloor$$

$$f^*(e) = f^*(u_{n-1}u_1^1) = \left\lceil \frac{2f(u_{n-1})f(u_1^1)}{f(u_{n-1})+f(u_1^1)} \right\rceil$$

or $\left\lfloor \dfrac{2f(u_{n-1})f(u_1^1)}{f(u_{n-1})+f(u_1^1)} \right\rfloor$

Hence the induced edge labelling can be found to be distinct and hence it can be claimed that the given graph G is Super Harmonic mean labelling graph.
Hence the proof.

Example.3.2: The 1-Join square of path P_4^2 is super harmonic mean graph



Theorem.3.3: If graph G is a M-Join square of path P_n^2 graph then

Number of Vertices in M-Join square of path P_n^2 graph = (M+1) (n) and

Number of edges in M-Join square of path P_n^2 graph = 2n (M+1)-(2M+3) for all n where M is the number of Joins in square of path P_n^2 graph of length n.

Proof: Let us prove the theorem by Method of Mathematical induction

Let us prove for the first positive integer i.e M=1, the number of joins in a square of path P_n^2 graph.

Given the graph G is 1- Join square of path P_n^2 graph then according to the construction of the 1--Join of square of path P_n^2 graph we have the number of vertices = 2n and the number of edges = 4n-5

Now for M=1

Number of vertices in 1-Join square of path P_n^2 graph = (M+1) (n) = 2n and hence it is true

Number of edges in 1-Join square of path P_n^2 graph =4n-5= 2n (M+1)-(2M+3)

Hence the induction holds good for M=1

Now let us assume that it is true for M=k, where k is the number of joins

Number of vertices in k-join square of path P_n^2 graph = (k+1) (n) and

Number of edges in k-join square of path P_n^2 graph = 2n (k+1)-(2k+3)

Now let us prove for M=k+1

i.e. M=k+1 means that the number of joins is k+1

We know that the number of joins k+1= number of k joins + number of 1 join of square of path P_n^2 graph.

We have assumed that it is true for k joins

Hence number of vertices in k+1 –join square of path P_n^2 graph = (k+1) (n) +2n= (k+3) n

Number of edges in k+1-join of square of path P_n^2 graph = 2n (k+1)-(2k+3) +2n-2

Which on simplifying

Number of edges in k+1-join of square of path P_n^2 graph = 2n (k+1)+2(n-k)-5

Hence the proof of induction. Therefore the theorem is true for M-Joins of square of P_n^2 graph.

Definition.3.4: Sum of the vertices of square of path P_n^2 graph is n^2 as the vertices are labelled with odd numbers and hence they form an arithmetic progression with common difference 2.

Theorem.3.5 : For any M-Join of square of path P_n^2 graph the Sum of the vertices of M- Join is given as a finite series $n^2 + \sum_{i=1}^{M} n(n+d_i)$ for M-joins, where n denotes the number of vertices and $d_1 = u_1^1 - u_1$, $d_i = u_1^i - u_1^{i-1}$ for $2 \le i \le M$.

Proof: Let G be a graph of square of path P_n^2 graph . We know the sum of the vertices of square of path P_n^2 is n^2 as the vertices are labelled with odd numbers. Now for each join of square of path P_n^2

graph we have to compute the sum of the vertices it can be understood that from the labelling procedure adopted for proving the 1-join of square of path P_n^2 graph that the difference between $d_1 = u_1^1 - u_1$ can be computed and hence the sum of 1- join square of path P_n^2 graph can be found to be $n^2 + n(n + d_1)$. Similarly for each join of square of path P_n^2 we compute the difference between $d_i = u_1^i - u_1^{i-1}$ for $2 \le i \le M$ and hence can be added to the sum of 1-join of square of path P_n^2 graph resulting in the sum of M-join of square of path P_n^2 graph to be $n^2 + \sum_{i=1}^{M} n(n + d_i)$. Hence the proof.

Corollary.3.6: For any M-Join of square of path P_n^2 graph sum of the each of the join added increases by $n(n + d_i)$ where $1 \le i \le M$ from the sum of the square of path P_n^2 graph whose sum is n^2

Proof: From Theorem.3.4 it can be seen that the sum of the vertices of M-Join of square of path P_n^2 graph is $n^2 + \sum_{i=1}^{M} n(n + d_i)$. From the proof of the theorem it is trivial to understand that each of the join increases the sum by $n(n + d_i)$ for $1 \le i \le M$. Hence the proof.

4. RESULTS

In this paper we have considered square of path P_n^2 graph and proved that it is super harmonic mean labelling graph and have identified a generalization method for M-joins of square of path P_n^2 graph

5. CONCLUDING REMARKS

We are investigating on the other types of similar graphs which can be also labelled so as to prove that they are super harmonic mean labelling graph

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